Double and Half angle Identities

Double Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

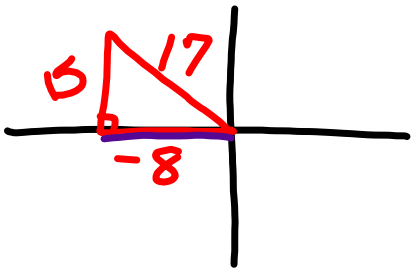
$$= 1 - 2\sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Given $\sin\theta = \frac{15}{17}, QII$

Find $\sin 2\theta, \cos 2\theta, \text{and } \tan 2\theta$



$\sin 2\theta = 2\sin\theta\cos\theta$

$2\left(\frac{15}{17}\right)\left(\frac{-8}{17}\right) = \frac{-240}{289}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\left(\frac{-8}{17}\right)^2 - \left(\frac{15}{17}\right)^2 = \frac{-161}{289}$

$\tan 2\theta = \frac{2\left(\frac{-15}{8}\right)}{\left(1 - \left(\frac{-15}{8}\right)^2\right)} = \frac{240}{161}$

Given $cos\theta = \frac{24}{25}, QIV$

Find $sin2\theta, cos2\theta, and\ tan2\theta$

$$\frac{2\tan 15^\circ}{1-\tan^2 15^\circ}$$

$\sin 540^\circ$

$\sin 2(270) = 2\sin 270 \cos 270$

$2 \cdot -1 \cdot 0$

0

$$\cos^2 15° - \sin^2 15°$$

$\theta = 15°$

$\cos 2(15)$

$\cos 30$

$\frac{\sqrt{3}}{2}$

Half-Angle Identities

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

± is determined by the Quadrant

Find $\cos 112.5^\circ$

$\cos \frac{225}{2}$

$-\sqrt{\frac{1+\cos 225}{2}}$

$-\sqrt{\frac{1+\frac{-\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2}{2}+\frac{-\sqrt{2}}{2}}{2}}$

$-\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \frac{2-\sqrt{2}}{2} \cdot \frac{1}{2} = \sqrt{\frac{2-\sqrt{2}}{4}}$

$-\frac{\sqrt{2-\sqrt{2}}}{2}$

$\frac{-\sqrt{2-\sqrt{2}}}{\sqrt{4}}$

$\sqrt{\frac{1}{2}\cdot\frac{\sqrt{2}}{4}}$

Given $tan\alpha = \frac{4}{3},\ 180° \leq \alpha \leq 270°$

Find $sin\frac{\alpha}{2}, cos\frac{\alpha}{2}, and\ tan\frac{\alpha}{2}$

HW